Automatic Streamlining for Constrained Optimisation

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Abstract. Augmenting a base constraint model with additional constraints can strengthen the inferences made by a solver and therefore reduce search effort. We focus on the automatic addition of *streamliner* constraints, which trade completeness for potentially very significant reduction in search. Recently an automated approach has been proposed, which produces streamliners via a set of streamliner generation rules. This existing automated approach to streamliner generation has two key limitations. First, it outputs a single streamlined model. Second, the approach is limited to satisfaction problems. We remove both limitations by providing a method to produce automatically a *portfolio* of streamliners, each representing a different balance between three criteria: how aggressively the search space is reduced, the proportion of training instances for which the streamliner admitted at least one solution, and the average reduction in quality of the objective value versus the unstreamlined model. In support of our new method, we present an automated approach to training and test instance generation, and provide several approaches to the selection and application of the streamliners from the portfolio. Empirical results demonstrate drastic improvements both to the time required to find good solutions early and to prove optimality on three problem classes.

Keywords: Constraint Programming · Streamliners

1 Introduction

An initial constraint model can be augmented through additional constraints. If well chosen, these constraints strengthen the inferences the solver can make and therefore reduce search. *Implied* constraints are inferred directly from the initial model and therefore do not alter the set of solutions to the model. Manual [15,16] and automated [7,9,17] approaches to generating implied constraints have been successful.

In contrast, *streamliner* constraints [20] (our focus herein) are not inferred from the initial model and often radically alter the set of solutions to the model in an attempt to focus effort on a highly restricted but promising portion of the search space. Streamliners trade the completeness offered by implied constraints for potentially much greater search reduction. They were originally derived manually by examining solutions of small instances of a problem class for patterns, which were used as the basis for streamliners [20,22,23,24]. For example Gomes and Sellmann added a streamliner requiring a latin square structure when searching for diagonally ordered magic squares [20].

More recently, an automated approach has been proposed, which produces streamliners via a set of streamliner generation rules [35,32] operating on the ESSENCE [12,13,14] specification of a problem class. Using training instances drawn from the problem class under consideration, streamliner candidates are evaluated automatically and the most promising ones are used to solve more difficult instances from the same problem class.

The existing automated approach to streamliner generation has two key limitations. First, it outputs a single streamlined model. If on a test instance this streamliner excludes all solutions the only remedy is to revert to the initial model. Second, the approach is limited to satisfaction problems. We remove both limitations by providing a method to produce automatically a *portfolio* of streamliners, each representing a different balance between three criteria: how aggressively the search space is reduced, the proportion of training instances for which the streamliner admitted at least one solution, and the average reduction in quality of the objective value versus an unstreamlined model.

In support of our new method, we present an automated approach to training and test instance generation, and provide several approaches to the selection and application of the streamliners from the portfolio. The result is the first automatic method to produce streamliners for optimisation problems and to offer alternatives if the most preferred streamliner is unsuccessful.

2 Candidate Streamliner Generation

As in [32], our approach proceeds from a specification of a problem class in the abstract constraint specification language ESSENCE [14], such as the SONET example in Figure 1. An ESSENCE specification comprises the problem class parameters (`given`); the combinatorial objects to be found (`find`); the constraints the objects must satisfy (`such that`); identifiers declared (`letting`); and an optional objective function (`min/maximising`). The key feature of the language is support for abstract decision variables, such as multiset, relation and function, as well as *nested* types, such as the multiset of sets in Figure 1.

The highly structured description of a problem an ESSENCE specification provides is better suited to streamliner generation than a lower level representation, such as a constraint modelling language like MiniZinc [27]. This is because nested types like multiset of sets must be represented as a constrained collection of more primitive variables, obscuring the structure that is useful to drive streamliner generation. We employ the same set of streamliner generation rules as [32], summarised in Table 1. High-order rules take another rule as an argument and lift its operation onto a decision variable with a nested domain such as the complex multi-set structure present in SONET. This allows for the generation of a rule such as enforcing that approximately half (with softness parameter) of the sets in the multiset only contain even numbers. Imposing extra structure in this manner can reduce search very considerably. Table 2 presents candidate streamliners automatically generated for the problem classes considered herein. Although rich, the set of ESSENCE type constructors is not exhaustive. Graph types, for example, are a work in progress [10]. At present, therefore, we might specify such a problem in terms of a set of pairs. The streamliner generator constraints would produce candidate streamliners based on this representation.

Using training instances drawn from the problem class under consideration, streamliner candidates are evaluated as follows. The CONJURE [1,3] automated modelling tool is used to refine the ESSENCE specification (including streamliner) into the solver-

```
$ SONET
given nnodes, nrings, capacity : int(1..)
letting Nodes be domain int(1..nnodes)
given demand : set of set (size 2) of Nodes

find network : mset (size nrings) of
            set (maxSize capacity) of Nodes
minimising sum ring in network . |ring|,
such that forAll pair in demand .
  exists ring in network . pair subsetEq ring

$ Minimum Energy Broadcast
letting dNodes  be domain int(1..n_nodes)
letting dDepths be domain int(1..n_nodes)
find parents: function (total)
                dNodes --> dNodes,
     depths : function (total)
                dNodes --> dDepths

$ Progressive Party
letting Boat be domain int(1..n_boats)
find hosts: set (minSize 1) of Boat,
     sched: set (size n_periods) of
              function (total) Boat --> Boat
```

Fig. 1: ESSENCE specifications for the three problem classes considered herein. Synchronous Optical Networking (SONET) [28] is given in full. For brevity, only the parameters and decision variable declarations (from which streamliners are generated) are shown for the Progressive Party Problem [33] and the Minimum Energy Broadcast Problem [6]

independent constraint modelling language ESSENCE PRIME, which SAVILE ROW [29] translates into input suitable for the constraint solver MINION [19].

3 Searching for a Streamliner Portfolio

Candidate streamliners are often most effectively used in combination [20]. In an attempt to find a single "best" streamlined model, Spracklen et al. described a Monte Carlo Tree Search [5] (MCTS)-based algorithm to search the lattice of models where the root is the original ESSENCE specification and an edge represents the addition of a streamliner to the combination associated with the parent node.

This search had a single objective, average search effort reduction across a set of training instances, which generates only one streamlined model per problem class. This model tends to achieve a high search effort reduction, but has difficulty generalising across the problem class. Furthermore, it is designed only for satisfaction problems. The optimisation problems with which Spracklen et al. experimented were converted into satisfaction problems by bounding the objective and searching for a satisfying solution. This is a serious limitation since a candidate streamlined model may find a solution quickly, but of poor quality, and may exclude the set of optimal solutions entirely.

Class	Trigger Domain	Name	Softness
First-order	`int`	odd{even}	no
		lower{upper}Half	no
	`function int --> int`	monotonicIncreasing{Decreasing}	no
		largest{smallest}First{Last}	no
	`function (X,X) --> X`	commutative	no
		associative	no
		non-commutative	no
	`partition from X`	quasi-regular	yes
	`sequence`	montonicIncreasing{Decreasing}	no
		largest{smallest}First{Last}	no
Higher-order	`matrix/set of X`	all	no
		most	yes
		half	no
		approxHalf	yes
	`function X --> Y`	range	no
		defined	no
		pre{post}fix	yes
		allBut	yes
	`function (X,X) --> Y`	diagonal	no
	`partition from X`	parts	no
	`sequence`	range	no
		defined	no

Table 1: The rules used to generate conjectures. Rows with a softness parameter specify a family of rules each member of which is defined by an integer parameter.

Problem	Streamliner Id	Description
Sonet	6	Exactly half the nodes installed on each ring are odd.
	13	Approx. half the nodes installed on each ring are odd.
	15	Approx. half the nodes on each ring are from the lower half of the Nodes domain.
	67	The objective variable is constrained to the lower half of its domain
MEB	18	Approx. half of the entries in the range of the parents function must be even
	41	The range of the depths function contains all odd entries
PPP	7	For half of the hosts the boats must be in the lower half of the Boats domain
	14	For approx. half of the hosts the Boats must be odd

Table 2: Sample streamliners generated for the three problem classes we consider (see Figure 1 for their ESSENCE specifications). References to odd/even are with respect to the integer identifiers associated with entities such as nodes or boats. Streamliner Id is a unique reference given to a streamliner when generated through CONJURE; we shall refer to these examples in Section 8.1

To address these problems we adopt a multi-objective optimisation approach, where each point x in the search space X is associated with a d-dimensional (d is the number of objectives) reward vector r_x in R^d. Our three objectives allow us explicitly to balance considerations of solution quality against how aggressively the streamlined model reduces search:

1. **Applicability**. The proportion of training instances for which the streamlined model admits a solution.
2. **Search Reduction**. The mean reduction in time to prove optimality in comparison with an unstreamlined model.
3. **Optimality Gap**. The mean percentage difference between the optimal value found by the streamlined model and the true optimal value under the unstreamlined model.

All objectives are transformed such that they can be maximized. With these three objectives for each streamliner combination we define a partial ordering on R^d and so on X using the Pareto dominance test. Given $x, x\prime \in X$ with vectorial rewards $r_x = (r_1, \ldots, r_d)$ and $r_{x\prime} = (r_{1\prime}, \ldots, r_{d\prime})$ r_x dominates $r_{x\prime}$ iff r_i is greater than or equal to $r_{i\prime}$ for $i = 1 \ldots d$.

To search the lattice structure for a portfolio of Pareto optimal streamlined models we have adapted the *dominance-based multi-objective MCTS (MOMCTS-DOM)* algorithm [34]. This has four phases, as summarised below and in Figure 2:

1. **Selection**: Starting at the root node, the Upper Confidence Bound applied to Trees (UCT) [5] policy is applied to traverse the explored part of the lattice until an unexpanded node is reached.
2. **Expansion**: Uniformly select and expand an admissible child
3. **Simulation**: The collection of streamliners associated with the expanded node are evaluated. The vectorial reward (Applicablity, Search Reduction, Optimality Gap) across the set of training instances is calculated and returned.
4. **BackPropagation**: The current portfolio; which contains the set of non dominated streamliner combinations found up to this point during search; is used to compute the Pareto dominance. The reward values of the Pareto dominance test are non stationary since they depend on the portfolio, which evolves during search. Hence, we use the cumulative discounted dominance (CDD) [34] reward mechanism during reward update. If the current vectorial reward is not dominated by any streamliner combination in the portfolio then the evaluated streamliner combination is added to the portfolio and a CDD reward of 1 is given, otherwise 0. Dominated streamliner combinations are removed from the portfolio. The result of the evaluation is propagated back up through all paths in the lattice to update CDD reward values, as shown in the figure.

4 Generating Diverse Training Instances

Our method relies on training instances from a given problem class to construct a high quality portfolio of streamlined models. Ideally these should be diverse, otherwise the portfolio may be skewed towards instances of one type and so not generalise across

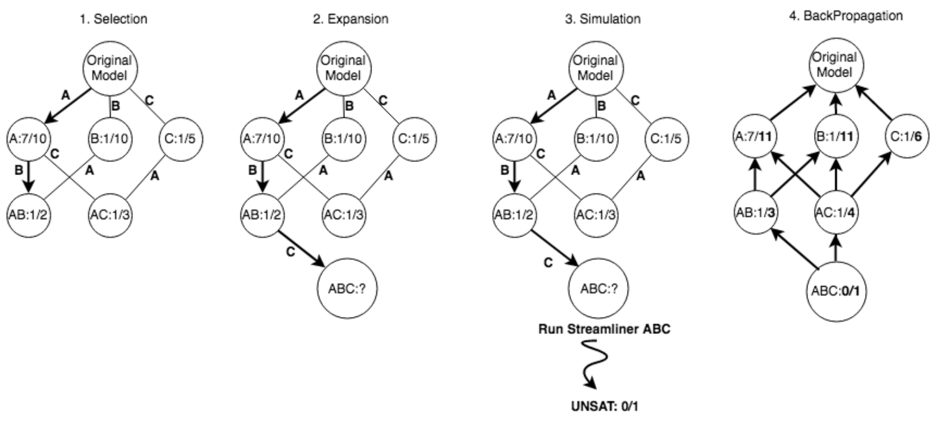


Fig. 2: MOMCTS-DOM operating on the streamliner lattice. A, B and C refer to single candidate streamliners generated from the original ESSENCE specification. As MOMCTS-DOM descends down through the lattice the streamliners are combined through the conjunction of the individual streamliners (AB, ABC). The nodes are labelled with *CDD reward value / times visited*.

the problem class. To ensure diversity, we employ an automated approach combining a per-class parameter generator and an algorithm configuration tool, described below.

For each problem class we wrote a simple instance generator that accepts a parameter setting and a random seed, and outputs a problem instance. At the moment the instance generator has to be manually created, and is the only part of the whole system that is not automated. However, this issue has been tackled in a recent work [2] within the same pipeline, which can be integrated into our system in the future. To keep the computational cost manageable, we require a set of relatively easy (but not trivial) instances for the training phase, which we define as solvable by MINION [19] on an unstreamlined model within a time limit of $[10, 300]$ seconds.

To find instances satisfying our criteria, the automatic algorithm configuration tool irace [25] is used. Parameters of each generator are tuned by irace with a performance measure guiding it towards regions of satisfiable instances within the required range of solving time. As the tuning procedure usually converges at certain regions of the search, multiple tunings with two settings of irace (the default and another that allows more exploration) are performed per problem class to obtain more diverse sets of instances.

There is an inherent tradeoff with the number of training instances used during search. If too few instances are used it diminishes the ability of the generated portfolios to generalise across the problem class, whereas a larger set reduces the iteration speed of MOMCTS to the point where it is ineffective in searching the streamliner lattice. Taking these considerations into account, for the experiments in this paper we have set the number of training instances to 50.

We first generate a large instance set using irace. Table 3 (column 2) presents the results of doing so for the problem classes we consider in this paper. In order to select our representative subset of 50 instances, instance-specific features are used to judge instance similarity. We use the features proposed in [18] and generated by MINION. All

Problem	Total Number Of Instances	Number Of Clusters
SONET	517	3
MEB	989	8
PPP	1264	8

Table 3: Instance Generation and Clustering. 50 training instances are selected from among the generated clusters.

features are normalised according to the z-score standardisation. GMeans clustering is used on the generated features to detect the number of instance clusters (see column 3 of Table 3). To build the training set instances are randomly selected from each cluster, with the number of instances taken from each weighted according to the relative size of each cluster.

The time limit for training instances, and the size of the training set are both parameters to our method, which will be investigated in future work.

5 Pruning the Streamliner Portfolio

As the number of objectives increases so, typically, does the size of the Pareto front, and hence the size of the generated streamliner portfolio. This is demonstrated in Table 4, which, in column 2, records the size of the streamliner portfolios generated through MOMCTS for our three problem classes. A large portfolio is cumbersome when considering streamliner selection and scheduling. We observed, however, that the streamlined models were not distributed evenly across the Pareto front. Therefore, GMeans clustering is used to identify the number of clusters present in the portfolio and a point from each cluster is then selected to form a representative subset of the full portfolio (see column 3 of Table 4).

Problem	Initial Portfolio Size	Pruned Portfolio Size
SONET	57	6
MEB	56	3
PPP	64	9

Table 4: We prune an initially generated streamliner portfolio through GMeans clustering and select a representative point from each cluster.

6 Selecting from the Streamliner Portfolio

Having constructed a streamliner portfolio for a particular problem class using MOMCTS and the set of training instances, for a given test instance the question arises as to

Algorithm 1 Lexicographic Streamliner Selection

```
procedure SELECTION(Portfolio P, Ordering, $Time_{total}$, Instance)
    P ← sort(P, by = Ordering)
    $Time_{Taken} \leftarrow 0$
    while $Time_{Taken} \leq Time_{Total}$ do
        Streamliner ← P.next()
        Stats ← Apply(Streamliner, Instance)
        if Stats→sat() then
            setBound(Instance, Stats.bound)        ▷ Set new bound on the instance
        end if
        $Time_{Taken} += $ Stats.time
    end while
end procedure
```

which streamlined models from the portfolio should be used, in what order, and according to what schedule. We consider both static lexicographic selection methods, which establish a priority order over our three objectives of Applicability, Search Reduction and Optimality Gap, and a dynamic method, which adjusts the selection based on the performance on the instance thus far.

6.1 Lexicographic Selection Methods

It is possible to order the streamlined models in a portfolio lexicographically by, for example, prioritising Applicability, then Search Reduction, and finally the Optimality Gap. Given three objectives, there are six such orderings to consider. Through preliminary testing it became apparent that only two of these orderings are effective, where the Applicability objective is prioritised. The other orderings trade Applicability for either Search Reduction or a better Optimality Gap. On more difficult test instances, significant search effort can be required to prove that an aggressive streamliner has rendered an instance unsatisfiable, which can lead to poor overall performance. Thus two lexicographic selection methods are used herein: {*Applicability First, Optimality Second, Reduction Third*} and {*Applicability First, Reduction Second, Optimality Third*}.

The selection process involves traversing the portfolio (using the defined ordering) for a given time period and applying each streamliner in turn to the given instance as shown in Algorithm 1. The schedule is static in that it only moves to the next streamlined model when the search space of the current one is exhausted. A key parameter is $Time_{total}$, which specifies the total budget in seconds for traversing the streamliner portfolio. In Section 8 for each selection method four different settings for this parameter are experimented with to explore its effect on overall performance.

6.2 UCB Streamliner Selection

During optimisation, typically a number of feasible solutions are discovered before the optimal objective value is found. This intermediate information can be used as an indicator of the performance of the streamlined model. For a given instance we have

Algorithm 2 UCBSelection

```
procedure SELECTION(Portfolio, Ordering, $Time_{total}$, Instance)
    $Time_{taken} \leftarrow 0$
    UCBTimeLimit ← 1
    NumberOfIterations ← 0
    Map                                          ▷ Mapping from Streamliner to Process
    while $Time_{taken} \leq Time_{total}$ do
        Streamliner ← UCTSelection(Portfolio)
        if Map[Streamliner].restart then
            Process ← remodel(instance, streamliner)        ▷ Remodel with the new bound
            Map[Streamliner].process ← Process
            Stats ← run(Process, UCBTimeLimit)
        else
            Process ← Map[Streamliner].process
            Stats ← run(Process, UCBTimeLimit)        ▷ Continue running existing process
        end if
        Map[Streamliner].visits += 1
        NumberOfIterations += 1
        if Stats→sat() then
            Map[Streamliner].reward += 1
            setBound(Instance, Stats.bound)        ▷ Set new bound on the instance
            for $S \leftarrow Map$ do
                if S != Streamliner then
                    Map[S].restart = True   ▷ New Bound was found; restart all other processes
                end if
            end for
        end if
        $Time_{taken} += $ Stats.time
    end while
end procedure
```

no prior knowledge of the suitability of a particular streamlined model and as such it is important to balance the time taken exploring the portfolio to identify the performance of each model while exploiting those that have already found solutions. Representing this as a multi-armed bandit problem allows us to employ well known regret-minimising algorithms to deal with the exploration/exploitation dilemma. The multi-armed bandit can be seen as a set of real distributions, each distribution being associated with the rewards delivered by one of the *K* levers. In our case this is the *K* streamlined models that comprise the portfolio. On each iteration a streamliner is selected to search the given instance and a reward is observed based upon the improvement to the objective value. The aim is at each iteration to apply the optimal streamliner, where optimality is defined as producing the largest increase/decrease in the value of the objective. The regret ρ after *T* rounds is defined as the expected difference between the reward sum associated with an optimal strategy and the sum of the collected rewards observed. The UCB1 [4] algorithm was chosen to solve the multi-armed bandit problem as first and foremost its regret grows logarithmically in line with the number of actions taken.

For each streamliner k we record the average reward x^k and the number of times k has been tried in the selection (n_j) out of a total of n iterations. On each iteration a streamliner is chosen that maximizes $x^k + \sqrt{2\log(n)/n_j}$. The reward distributions for an individual streamliner are not fixed, so this is not a Stationary Multi-Armed Bandit problem. However, if a streamliner performs well, we expect it will continue performing well during search even if there is a slight variation in the mean reward. We have found that using UCB1 gives good results. Future work could investigate the use of Upper Confidence Bound policies for non-stationary bandit problems, such as the family of Exp3 algorithms [21,26].

When traversing the portfolio UCB performs incremental evaluation, it runs a streamliner for a set time, observes the results, and potentially moves on before the corresponding search space has been exhausted. When the streamliner is pre-empted it is necessary to pause the search in order to avoid repeating work if it is rescheduled at a later point. The only exception to this is whenever a new bound on the objective is discovered all of the streamliners from the portfolio, aside from the current streamliner, are restarted and remodeled with the new bound. There are two main benefits to doing this. Firstly, by restarting the streamliner has the newly constrained bound at the top of the search tree which allows it to make more informed decisions higher up without descending into unsatisfactory subtrees. Secondly, by remodeling it takes advantage of the toolchain (CONJURE and SAVILE ROW) which may be able to reformulate the model based upon this new information and produce reductions at the solver level. Algorithm 2 shows the UCBSelection process in detail.

7 Experimental Setting

We evaluate our automated streamlining approach on the three problem classes in Figure 1. We selected these problems to give good coverage of the abstract domains available in ESSENCE, such as set, multi-set and function. Furthermore, SONET and Progressive Party have nested domains: multi-set of set and set of function respectively.

Our hypothesis is that a streamliner portfolio, generated automatically on a set of automatically generated training instances from a given problem class, can be employed to solve more difficult test instances to deliver substantial performance improvements relative to an unstreamlined model. Training instances were generated as per Section 2, with a time limit of $[10, 300]$ seconds. Test instances are generated using the same instance generator and the tuning tool irace but with a time limit of $(300, 3600]$ seconds. 50 instances are selected randomly to form the test set.

Care must be taken when considering the proof of optimality of our test instances. Although in solving a streamlined model the constraint solver may exhaust the search space this is not a proof that the current objective value is optimal. This is because streamliners are not necessarily sound, hence a streamlined model may exclude the set of optimal solutions. For this reason, after the streamliner portfolio has been run for its allotted time, we use the remainder of the time budget to run the unstreamlined model, starting from the best objective value found by the streamliner portfolio, to provide the optimality proof. The benefit of streamlining in this context is in finding high quality solutions much more quickly than the unstreamlined model.

All experiments were run on a cluster of 280 nodes, each with two 2.1 GHz, 18-core Intel Xeon E5-2695 processors. MOMCTS was run on a single core with a budget of 4 CPU days for each problem class. Results on 50 test instances under the unstreamlined and streamlined models are reported, where every test instance was run with three random seeds.

Source code, instance generators, datasets and detailed results are available at https://github.com/stacs-cp/CP2019-Streamlining.

8 Results

Table 5 summarises results on 50 test instances (3 runs/instance) for each of our three problem classes. We evaluate four different approaches: an unstreamlined model, and streamliner portfolios with UCB selection, lexicographic ordering {*Applicability First, Optimality Second, Reduction Third*} (denoted *opt-second*), and lexicographic ordering {*Applicability First, Reduction Second, Optimality Third*} (denoted *red-second*). For each streamliner selection method, a parameter is the amount of time allocated to the streamliner portfolio before handing over to the unstreamlined model to prove optimality. Four different values for this time budget were tested: 30, 60, 120 and 300 seconds.

Results in Table 5 are strongly positive. They show that all the streamliner portfolio approaches can not only find an optimal solution and prove optimality on more test instances than the unstreamlined model, but also vastly reduce the amount of time required for both tasks. In general, the UCB-30s variant has the best overall performance across the three problem classes, and provides consistently robust improvement over the unstreamlined model.

Figure 3 presents more details of how the streamliner approaches improve on the unstreamlined models on an instance basis. In these plots, we use the time-reduction ratio, a "normalised" version of the speed-up values reported in Table 5 for presentation: as the speed-up values can be arbitrarily large, many data points in the speed-up plots can appear in a very small range, making them difficult to distinguish. The reduction ratio, which is calculated as $1 - 1/\mathit{speed\text{-}up}$, is limited to at most one and can be easily scaled. For brevity, we only show in Figure 3 results of the streamliner variants with the time limit of 30 seconds. Each data point corresponds to a pair of instances and random seeds. The plots show that the solving time of the test instances are well distributed across the x-axis, which is a good indication for the diversity of the test instance set. There are several cases where the unstreamlined model cannot find or prove optimality within the time budget and the streamliner can, which are represented by the data points on the rightmost side after the vertical red lines.

The MEB results demonstrate strong performance of all three streamliner approaches on all test instances. On SONET, UCB-30s clearly has better performance compared with the other two approaches, which aligns with the summary results in Table 5. While still strongly positive, on PPP the reduction provided by the streamliner approaches is not quite as strong as for the other two problem classes. There are a minority of cases where even the best streamliner approach, UCB-30s, cannot find or prove optimality within the time budget, as shown by the data points in the bottom-right corners.

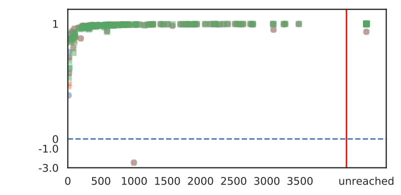
	Strategy	mean #proved (1-hour)	Finding an optimal solution						Finding and prove optimality					
			time(s)			speed-up ratio			time(s)			speed-up ratio		
			p10	p50	p90	p10	p50	p90	p10	p50	p90	p10	p50	p90
MEB	unstreamlined	35	157.9	1185.2	13893.9				311.1	1976.2	16781.3			
	UCB-30s	**50**	6.1	8	**11.0**	14.2	158.2	1583	15.2	22.2	**176.7**	6.6	43.6	492.2
	UCB-60s	**50**	4.4	7.2	12	15	150.3	1552.2	16.1	24.6	188.7	6.9	35.6	521.9
	UCB-120s	**50**	4.5	7.8	12.1	14.9	158	1604	15.1	24.8	220.9	6.2	36.1	518.4
	UCB-300s	**50**	4.5	7.1	12.1	15	157.5	1605.4	14.9	24.9	345.1	5.2	32.1	416.6
	opt-second-30s	49.7	**4.1**	6.3	13.4	14.1	171.1	1701.5	11.6	22.9	221.3	7.3	44.6	605.9
	opt-second-60s	49.7	**4.1**	6.6	14.9	15.7	174.3	1833.4	11.7	22.5	199.6	7	45.3	625.4
	opt-second-120s	**50**	4.2	6.2	13.6	**19.9**	178.3	1776.7	11.7	21.8	181.6	7.3	46.5	594.9
	opt-second-300s	**50**	**4.1**	**6.1**	12.8	**19.9**	170.9	1865.8	**11.5**	21.8	176.9	7.5	**47.6**	**647.0**
	red-second-30s	49.7	**4.1**	6.7	13.6	14.1	156	1845.1	11.8	22.8	249.1	7.3	43	532.2
	red-second-60s	49.7	4.2	**6.1**	12.8	15.3	**187.0**	1878.3	11.8	**21.7**	198.7	7.3	45	646.3
	red-second-120s	**50**	**4.1**	6.2	12.6	16.9	177.4	**1903.5**	11.6	22.1	178.1	7.2	46.3	605.4
	red-second-300s	**50**	**4.1**	**6.1**	13.5	16.8	167.5	1891	11.7	22.3	178.8	**7.6**	47.5	625.1
PPP	unstreamlined	41.3	73.4	564.3	3123				313	1339.7	6908.1			
	UCB-30s	47.7	13	**73.7**	**1007.9**	**1.2**	**4.1**	52	49.2	**350.8**	1946.6	**1.0**	**3.0**	**29.3**
	UCB-60s	**48.3**	19.2	105.9	1078.7	0.9	2.9	28.7	86.1	428.8	2141.5	0.9	2.5	24.4
	UCB-120s	**48.3**	18.9	163.3	1129.7	0.7	2.5	31.8	135.5	449.6	**1936.2**	0.9	2.1	16.8
	UCB-300s	**48.3**	19	344.6	1311.3	0.4	1.6	30.1	323.9	646.3	2273.2	0.6	1.4	10.5
	opt-second-30s	46.7	8.3	105.1	1340.5	0.9	3.5	75.1	**44.1**	419.4	2592.5	0.9	2.4	26.2
	opt-second-60s	47	**8.1**	105.8	1444.2	0.8	3.4	75.2	73.7	453.5	2640.3	0.8	2.3	18.9
	opt-second-120s	47.3	8.9	142.9	1765.1	0.7	3.6	**76.5**	113.1	486	2716.7	0.8	1.9	17.6
	opt-second-300s	47.7	8.9	211	1349.3	0.5	3.1	72.4	110.8	599.1	2703.2	0.7	1.8	15.5
	red-second-30s	45	14.7	177.7	2344.7	0.7	2	18.9	73.3	626.2	3537.7	0.8	1.7	14.8
	red-second-60s	45.3	21.2	195.2	2341.6	0.6	2.1	15.6	96.1	643.2	3174.7	0.7	1.8	13.8
	red-second-120s	45.7	13.6	175.7	2384	0.6	2.1	17.5	136.5	591.5	3095	0.6	1.8	11.1
	red-second-300s	45.3	13.6	228	2731.5	0.6	1.9	16.8	157	657.6	3339.1	0.6	1.4	8.4
SONET	unstreamlined	43	539.5	1263.2	3820.3				574.4	1417.8	3954			
	UCB-30s	**50**	5	21.8	**121.9**	**10.3**	49.7	341.5	34	**42.3**	**174.0**	**6.6**	**23.4**	60.5
	UCB-60s	**50**	6.1	28	131.9	8.5	38.1	300.3	63.3	75.3	198.7	4.9	14.4	42.1
	UCB-120s	46	6	31.1	246.8	3.4	31.5	321.9	121.2	132.2	581.2	2.3	7.6	32.1
	UCB-300s	**50**	7	30.7	344.5	3.8	33.4	287.3	111.8	310.8	437.8	1.7	4.2	22.9
	opt-second-30s	49.3	3.5	9	1023.8	1.4	112.7	553.9	27.7	72.7	1023.2	1.4	19.2	**70.5**
	opt-second-60s	49.7	3.5	9	443.2	1.5	113.1	611.4	27.6	93.6	644	1.5	15.9	66.9
	opt-second-120s	49.3	3.3	**8.3**	455.6	1.3	117.3	**677.9**	**26.9**	120.7	701	1.3	14.6	68.6
	opt-second-300s	49.3	3.7	8.4	549	3.6	**121.0**	549.9	28	123.1	770.6	1.1	10.3	69.4
	red-second-30s	47.7	**3.0**	115.4	1749.6	0.8	10.6	483.5	27.7	227.3	2167.4	0.8	5.2	61.7
	red-second-60s	47.7	**3.0**	105.3	1760.9	0.8	14.2	530.9	28.1	185	2137.2	0.8	7.2	64.1
	red-second-120s	47.3	**3.0**	96.7	1532.5	0.8	16	506.3	28.3	157.8	2295.6	0.8	7.6	62.6
	red-second-300s	47.7	**3.0**	96	1451.4	0.9	18.2	533.8	27.1	221.4	1717.6	0.8	6.1	65.2

Table 5: Summary results on 50 test instances (3 runs/instance) on three optimisation problem classes: MEB, PPP and SONET. The first column, mean #proved 1-hour, represents the average number of instances solved within one hour. All streamliner portfolio variants significantly outperform the unstreamlined model by this simple measure. The remaining columns report results where each run is now given a maximum amount of 96 CPU-hours (as tuning and generation of test instances is performed on the basis of one seed, on the two other seeds it is possible for the unstreamlined model to time out at one CPU hour). They include the time to reach an optimal solution, the time to both reach an optimal solution and prove its optimality; and the corresponding speed-up ratios when compared to the unstreamlined model. For each measurement, we report the 10^{th} percentile (p10), the median (p50), and the 90^{th} percentile (p90). These values are reported as the mean can be skewed by outliers. In particular, if the optimal solution is not proved this results in a large time value (96 hours = 345600 seconds) for that run. The percentiles avoid this situation and show a clearer overall trend.

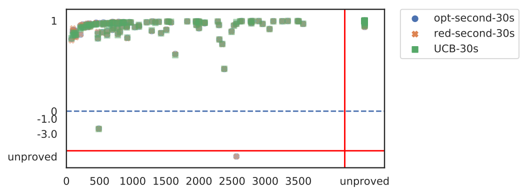
Table 5 and Figure 3 demonstrate that the time to prove optimality is very significantly reduced through the application of streamliners. This stems from their ability to find high quality feasible solutions quickly. Hence, once the time allocated to the streamlined models has elapsed, the unstreamlined model begins from an optimal or very high quality objective value, requiring much less effort to exhaust the search space.

8.1 UCB Streamliner Selection: Discussion

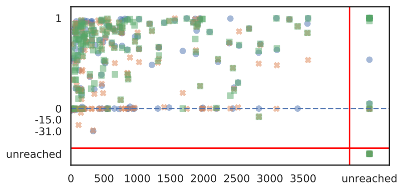
In this section, we discuss the UCB approach for streamliner selection in more detail, as UCB-30s achieves the best overall performance across the three problem classes, both in terms of reduction to finding the optimal objective value and reduction to proving



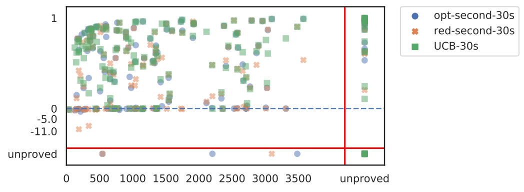
(a) MEB - time to optimal



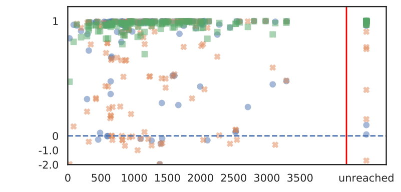
(b) MEB - time to proof



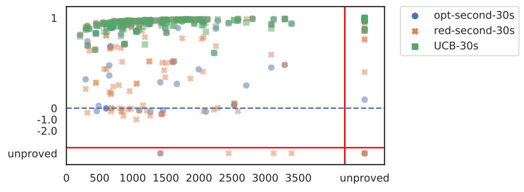
(c) PPP - time to optimal



(d) PPP - time to proof



(e) SONET - time to optimal



(f) SONET - time to proof

Fig. 3: Reduction ratio of streamliner methods with 30 seconds for scheduling of the streamliner portfolio. Two reduction ratio values are reported: reduction in time to reach an optimal solution, and reduction in time to reach an optimal solution and prove its optimality. The x-axis represents the time required by the unstreamlined model. The y-axis shows the the reduction value. Each data point corresponds to a pair of (instance, random seed). These plots focus on the region within a 1-hour time limit: all data points outside that ranges are shrunk into the same region. More specifically, runs where the (unstreamlined model) streamliner methods do not reach an optimal solution or does not prove optimality in one hour are separated by the red (vertical) horizontal lines. The reduction values, however, are still the true values calculated based on the 4-day CPU limit. As most data points lie within the range of $y \in [0, 1]$, the plot is rescaled so that this range is zoomed in for better visualisation.

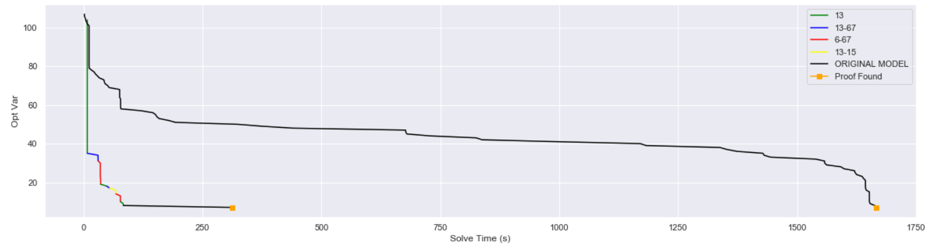


Fig. 4: Objective value progression from the unstreamlined model compared with its progression under the UCB selection method for a representative SONET instance.

optimality. In contrast to the lexicographic methods, which only move on to the next streamlined model when the search space of the current one is exhausted, UCB benefits from its ability to sample the entire streamliner portfolio. After the initial exploration phase, where each streamliner is given its initial application, UCB then selects streamliners based upon the observed rewards. Its main advantage is the ability to balance the exploration and exploitation of the streamlined models in the portfolio.

It is not always the case that the objective is found purely through the application of one streamliner. For SONET, on average three streamliners are used across the 50 test instances to arrive at the optimal objective value. Access to the whole portfolio allows UCB to descend upon the optimal objective value more quickly and is one reason for its success. The application of several different streamliners at different time points can be used to reduce the bound of the objective in an effective manner as per Figure 4.

The UCB algorithm exploits the streamliners that have previously been shown to produce an improvement in the objective value. This can be very clearly shown from Figure 4 where for an instance from SONET the streamliners 13, 13-67 and 6-67[1] (explained in Table 2) improve the objective multiple times during the course of the selection process. This is due to the fact that UCB is continuing to exploit those streamliners as previously they had success. However, it is also crucial to continually explore the portfolio in an attempt to find streamliners that did not initially have success but may do after a certain number of iterations. Streamliner 13-15 is an example of such a case.

8.2 Time Allocated to the Streamliner Portfolio: Discussion

From Table 5 it can be seen that the $Time_{Total}$ parameter as defined in Algorithms 1 and 2 can have a large impact on the overall performance of the selection method. There is a general trend (excluding MEB which will be discussed separately) that as the $Time_{Total}$ increases the time both to find and prove the optimal objective value increases. This may seem puzzling initially: if using a $Time_{Total}$ of 30s reduces the time to find the optimal objective value to a certain extent, it might be expected that a $Time_{Total}$ of 300s will do equally well. However, there are two things to consider. First, streamliners from the portfolio are not guaranteed to preserve the optimal value and so there is the potential for an optimality gap between what the streamliners can

[1] 13-67, for example, indicates a streamlined model including both streamliner 13 and 67

find and the true optimal of the instance. Therefore, the true optimal is only found after the switch to the unstreamlined model occurs. Second, on average the streamliners converge upon their optimal value in a very short period of time, 17s, 7s and 12s for SONET, MEB and PPP respectively. By increasing the $Time_{Total}$ parameter it delays the point at which the switch occurs to the unstreamlined model which in turn delays the point at which the true optimal is found. However, for MEB the $Time_{Total}$ does not have a large impact on performance and this is due to the fact that the streamliners in the portfolio generally exhaust their search space very quickly. Hence, the whole portfolio can be traversed before $Time_{Total}$ is reached and so the time at which the switch to the unstreamlined model occurs is generally the same across all parameter settings.

The increase in time to prove optimality occurs as if the T_{total} parameter is set too large then when the optimal value is found at time T_{opt}, the whole duration from $T_{opt} \rightarrow T_{total}$ is spent proving the optimality of that solution in the streamlined subspaces. Since proving optimality with respect to the streamliners does not prove optimality on the unstreamlined model and so the whole time from $T_{opt} \rightarrow T_{total}$ is wasted.

9 Conclusion and Future Work

We have presented the first automated approach to generating streamliners automatically for optimisation problems, and for their selection and scheduling when employed on unseen instances. On three quite different problem classes the results are very encouraging, with vastly reduced effort both to find and to prove optimal objective values.

An important question we plan to investigate further is the applicability of our method to identify in which contexts our streamliner can and cannot help. In the context of optimisation the benefit of streamlining lies in the early identification of the optimal, or at least high quality, values for the objective. Where an unstreamlined model is able to identify the optimal value quickly, the benefit of streamlining will be limited. When considering satisfaction problems, however, streamlining can be used throughout the search and we will compare the portfolio approach developed herein with the single selection provided by the method presented in Spracklen et al. [32].

Furthermore, there are several methods for devising good search strategies for constrained optimisation problems. Recent research suggest using machine learning to design a promising search ordering [8], using solution density as a heuristic indicator [31] and a number of value ordering heuristics to find good solutions early [30,11]. Streamlining constraints can potentially be used in combination with the existing methods for devising good variable and value selection heuristics to achieve even better results.

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